Extending Catmull-Clark Subdivision and PCCM with Polar Structures

Ashish Myles  Kestutis Karčiauskas  Jörg Peters

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Overview
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K. Karčiauskas
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Polar Structure Examples
Modeling
Refinement
Polar refinement
General mesh refinement
NURBS Constructions
Overview
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Valence $\neq 4$
Overview

Catmull-Clark subdivision

Valence ≠ 4

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Modeling

Refinement
Polar refinement
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NURBS
Constructions
Overview

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PCCM
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New Bi-cubic polar subdivision

Polar layout

Catmull-Clark subdivision

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Overview

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New NURBS capping

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Catmull-Clark subdivision

Valence ≠ 4

PCCM
Polar structures appear naturally

Eye courtesy of "Blender: Noob to Pro"
Remove those unsightly wrinkles

Catmull-Clark  Our method
Make predictable ripples

Catmull-Clark

Our method
Model designers face the following challenges

**Conventional picture**

1. Align control mesh along features.

2. Use only quads.

Model designers face the following challenges

**New and improved picture**

1. Align control mesh along features.

2. Use quads *and* polar structures.

3. Keep quad-mesh valence low. High polar valence OK!
Model a face using polar structures and multi-sided
Model a face using polar structures and multi-sided
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Modeling with polar connectivity

1. keeps the Catmull-Clark valence low,
2. shifts high-valence connectivity to polar structures, and
3. orients the control lines along model features (e.g. mouth).
Mesh Refinement

- Bi-cubic subdivision
- Polar layout
- NURBS capping

- Catmull-Clark subdivision
- Valence $\neq 4$
- PCCM

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Polar Structure
Examples
Modeling
Refinement
Polar refinement
General mesh refinement
NURBS Constructions
Polar refinement

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Polar Structure
Examples
Modeling
Refinement
Polar refinement
General mesh refinement
NURBS
Constructions
Polar refinement

\[
\begin{align*}
\alpha &= \beta - \frac{1}{4}, \quad \beta := \frac{5}{8}, \\
\gamma_k &= \frac{1}{n} \left( \beta - \frac{1}{2} + \frac{5}{8} c_n^k \right) + (c_n^k)^2 + \frac{1}{2} (c_n^k)^3
\end{align*}
\]
Polar refinement

\[
\alpha := \beta - \frac{1}{4}, \quad \beta := \frac{5}{8},
\]

\[
c^n_k := \cos \left( \frac{2\pi k}{n} \right),
\]

\[
\gamma_k := \frac{1}{n} \left( \beta - \frac{1}{2} + \frac{5}{8} c^n_k \right) + \left(c^n_k\right)^2 + \frac{1}{2} \left(c^n_k\right)^3
\]
Polar is easily combined with Catmull-Clark
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Polar Structure
  Examples
  Modeling

Refinement
  Polar refinement
  General mesh refinement

NURBS Constructions
Polar is easily combined with Catmull-Clark Subdivision and PCCM with Polar Structures

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Polar Structure
Examples
Modeling
Refinement
Polar refinement
General mesh refinement

NURBS
Constructions
Polar is easily combined with Catmull-Clark
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Polar is easily combined with Catmull-Clark

$\Rightarrow C^1$ with bounded curvature at the polar limit point.

- Verified using standard analysis tools from subdivision theory.
Results
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Polar Structure
Examples
Modeling
Refinement
Polar refinement
General mesh refinement
NURBS Constructions

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Bi-cubic subdivision

Polar layout

NURBS capping

NURBS Constructions

Catmull-Clark subdivision

Valence $\neq 4$

PCCM
Polar structures can be $C^1$ capped by a single NURBS patch
Polar structures can be $C^1$ capped by a single NURBS patch.

1. subdivide radially

$k$-times subdivided mesh

original control mesh
Polar structures can be $C^1$ capped by a single NURBS patch

1. subdivide radially

2. project

$k$-times subdivided mesh

original control mesh

periodic B-spline

original control mesh

periodic B-spline
Capping Polar with a single NURBS patch
Capping Polar with a single NURBS patch
Capping Polar with a single NURBS patch
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Polar Structure
Examples
Modeling
Refinement
Polar refinement
General mesh refinement
NURBS Constructions
Capping Polar with a single NURBS patch

$\Rightarrow C^1$ with bounded curvature at the polar limit point.

- Singular parametrizations typically tricky
- Our B-spline patch $=$ limit surface of a particular subdivision scheme
- Analyze using subdivision machinery!
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Polar Structure
Examples
Modeling
Refinement
Polar refinement
General mesh refinement

NURBS Constructions

Conclusion

Bi-cubic subdivision

Polar layout

NURBS capping

Catmull-Clark subdivision

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Questions?

Bi-cubic subdivision

Polar layout

NURBS capping

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PCCM

Questions?
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Examples
Modeling
Refinement
Polar refinement
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NURBS
Constructions
High-valent Catmull-Clark layout $\rightarrow$ polar layout
High-valent Catmull-Clark layout → polar layout
Analysis

\[ A = \begin{bmatrix} A_0 & A_1 & \ldots & A_{n-1} \\ A_{n-1} & A_0 & \ldots & A_{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ A_1 & \ldots & A_{n-1} & A_0 \end{bmatrix} \]

\[ A_0 := \begin{bmatrix} 1/n & 0 & 0 & 0 \\ 1/n & \Gamma_0 & 0 & 0 \\ 0 & 3/4 & 1/4 & 0 \\ 0 & 3/16 & 11/16 & 1/8 \end{bmatrix}, \quad A_i := \begin{bmatrix} 1/n & 0 & 0 & 0 \\ 1/n & \Gamma_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \]
Analysis

Spectral analysis of $A$ (circulant) gives desired properties:

 Eigenvalues: $1, \frac{1}{2}, \frac{1}{2}, 1/4, \ldots, 1/4$.

Characteristic map is regular ($\Rightarrow C^1$).

Geometric multiplicities $=$ algebraic multiplicities for $A$.

$\Rightarrow$ Bounded curvature.
Tensor product B-spline refinement
Separating Catmull-Clark and polar extraordinary limit points

\[ 1 - \alpha \frac{\alpha}{n} \]
\[ 1 - \beta \gamma_i \]
\[ \gamma_{i+1} \]
\[ \gamma_{i-1} \]

radial

circular

\[ \frac{1}{8}, \frac{6}{8}, \frac{1}{8} \]
\[ \frac{1}{2}, \frac{1}{2} \]
\[ \frac{1}{8}, \frac{6}{8} \]
\[ \frac{1}{2}, \frac{1}{2} \]