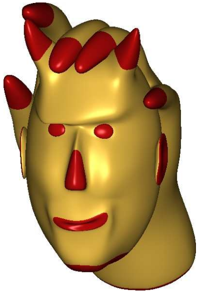
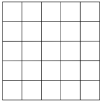
Extending Catmull-Clark Subdivision and PCCM with Polar Structures

Ashish Myles   Kestutis Karčiauskas   Jörg Peters



Pacific Graphics
November 1, 2007

Overview



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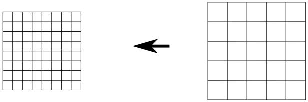

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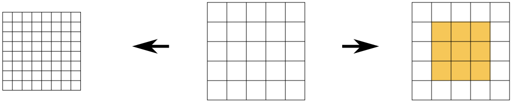

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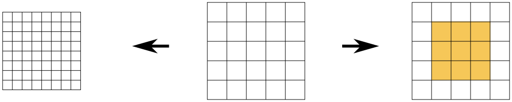
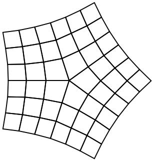
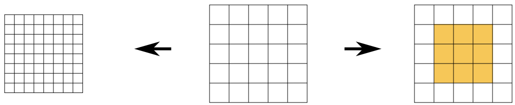
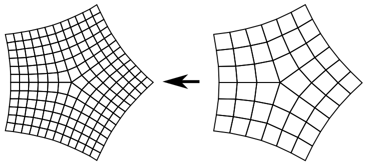




Valence $\neq 4$

Overview

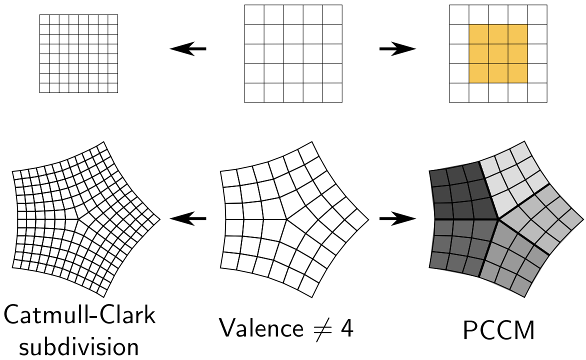




Catmull-Clark subdivision

Valence $\neq 4$

Overview



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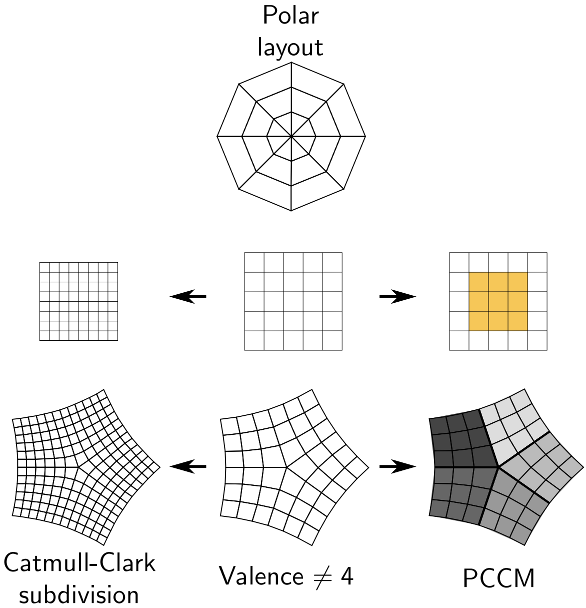
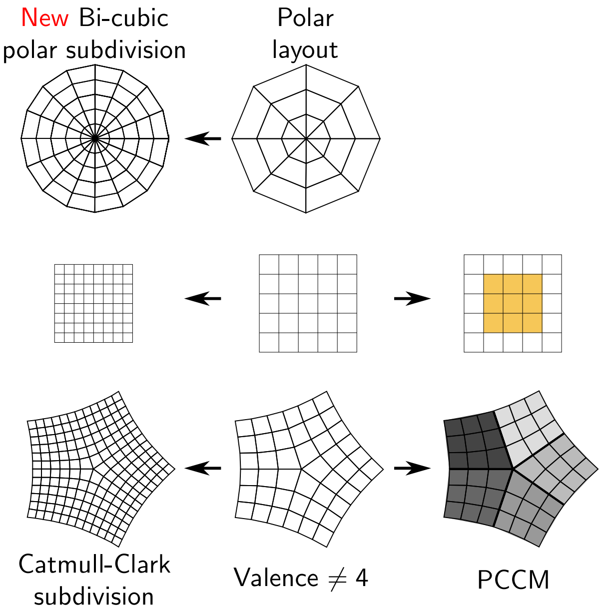
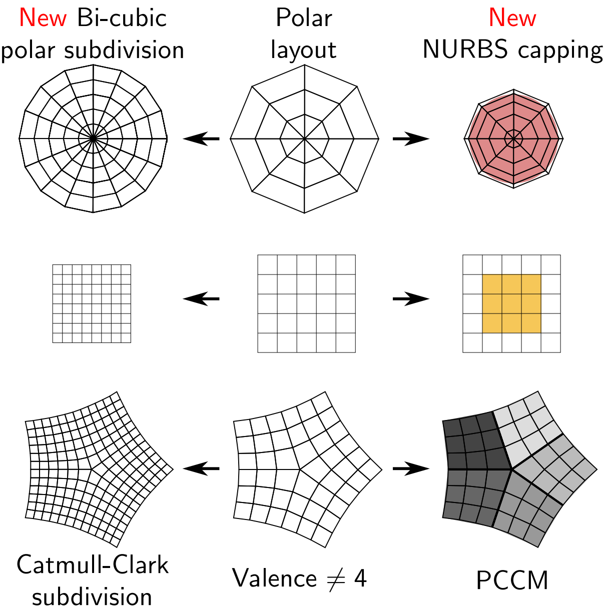


Overview

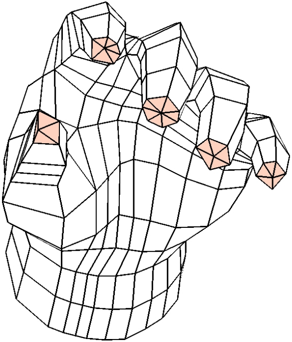


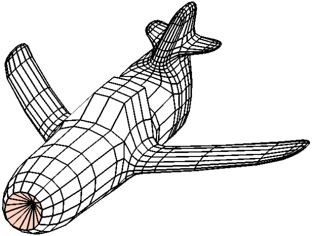
New Bi-cubic polar subdivision
Polar layout
Catmull-Clark subdivision
Valence $\neq 4$
PCCM

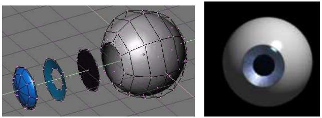
Overview


New Bi-cubic
polar subdivision
Polar
layout
New
NURBS capping
Catmull-Clark
subdivision
Valence $\neq$ 4
PCCM

Polar structures appear naturally

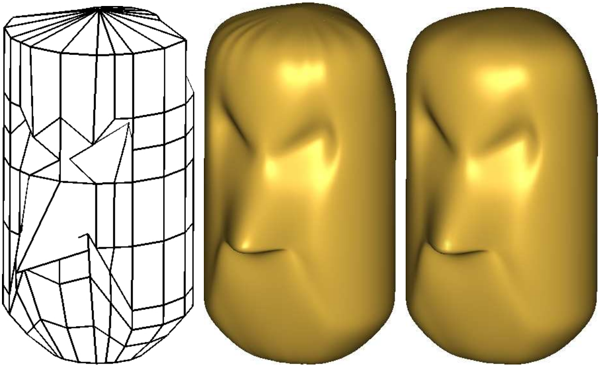






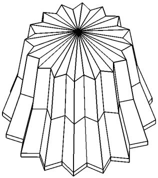
Eye courtesy of "Blender: Noob to Pro"

Remove those unsightly wrinkles



Catmull-Clark Our method

Make predictable ripples



Catmull-Clark

Our method

Model designers face the following challenges

Conventional picture

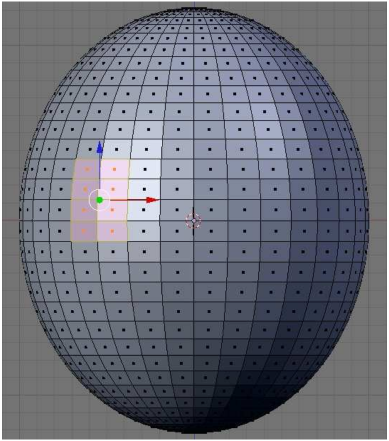
1. Align control mesh along features.
2. Use only quads.
3. Keep valence low.

Model designers face the following challenges

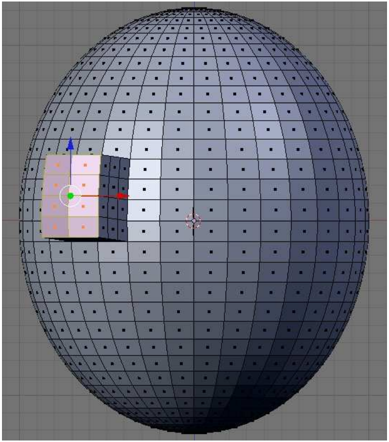
New and improved picture

1. Align control mesh along features.
2. Use quads *and* polar structures.
3. Keep quad-mesh valence low. High polar valence OK!

Model a face using polar structures and multi-sided



Model a face using polar structures and multi-sided



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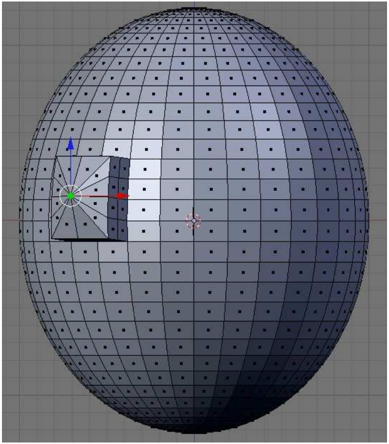

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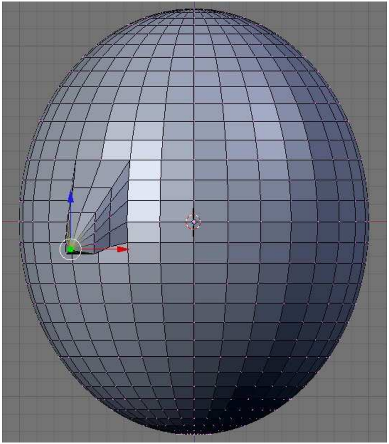

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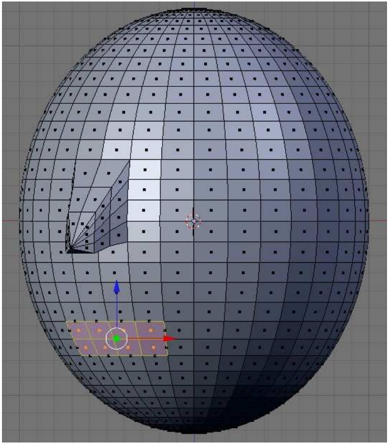

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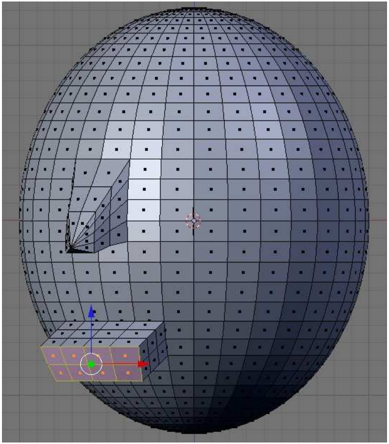

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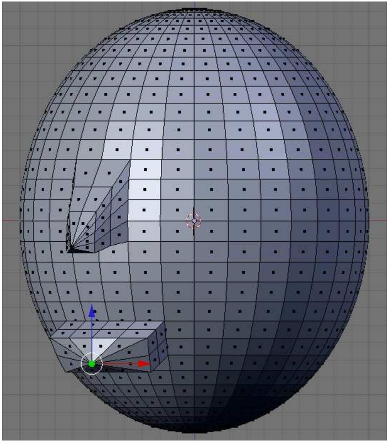

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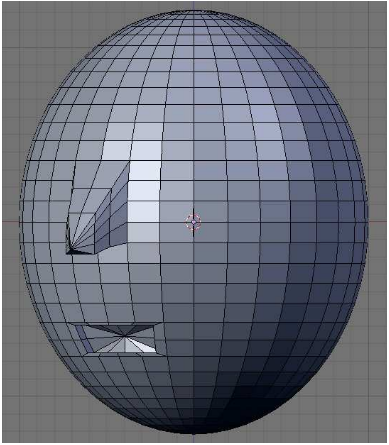
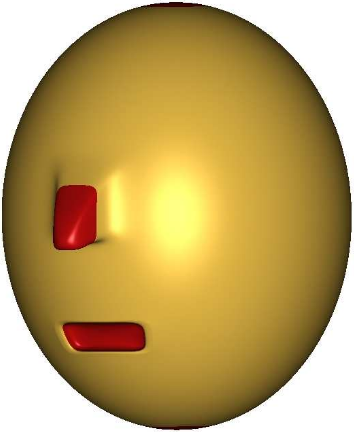
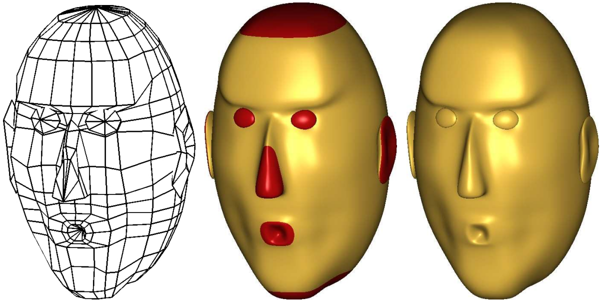


Model a face using polar structures and multi-sided

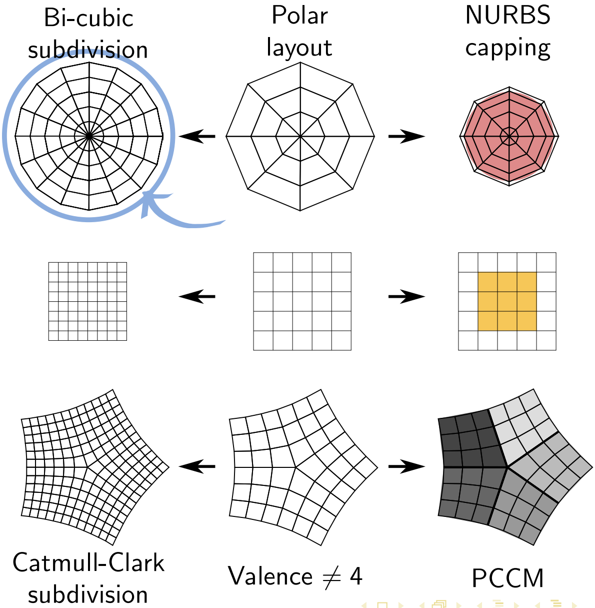


Modeling with polar connectivity

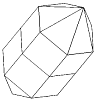


1. keeps the Catmull-Clark valence low,
2. shifts high-valence connectivity to polar structures, and
3. orients the control lines along model features (e.g. mouth).

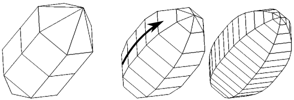
Mesh Refinement



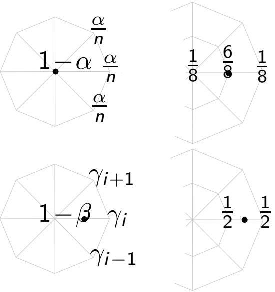
Polar refinement



Polar refinement



radial



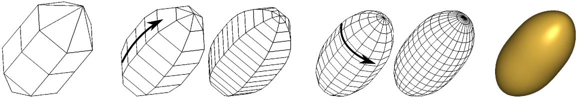
radial

$$\alpha := \beta - \frac{1}{4}, \quad \beta := \frac{5}{8},$$

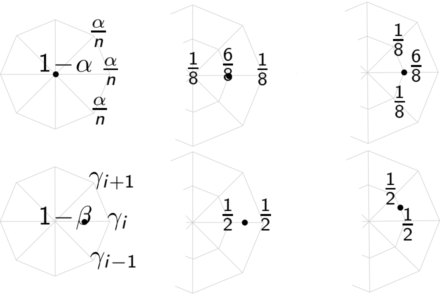
$$c_n^k := \cos\left(\frac{2\pi k}{n}\right),$$

$$\gamma_k := \frac{1}{n}\left(\beta - \frac{1}{2} + \frac{5}{8}c_n^k + (c_n^k)^2 + \frac{1}{2}(c_n^k)^3\right)$$

Polar refinement



radial circular



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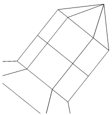
Polar is easily combined with Catmull-Clark



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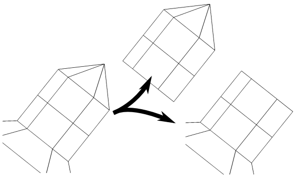

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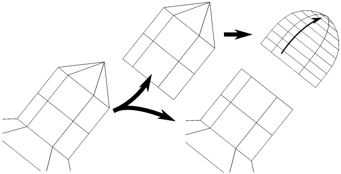

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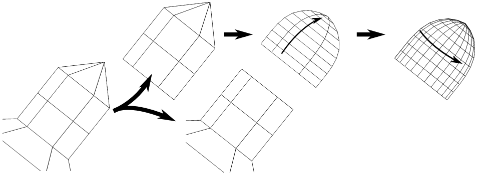

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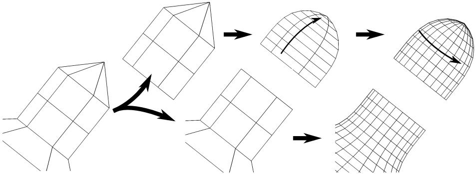

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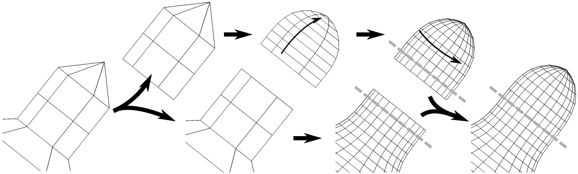

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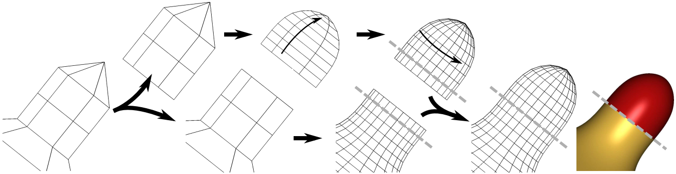
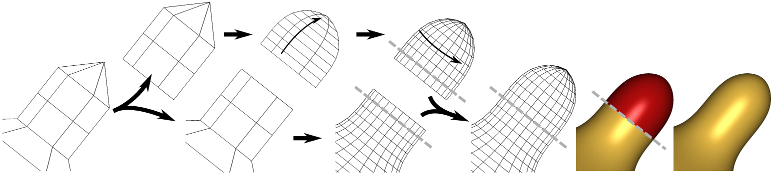


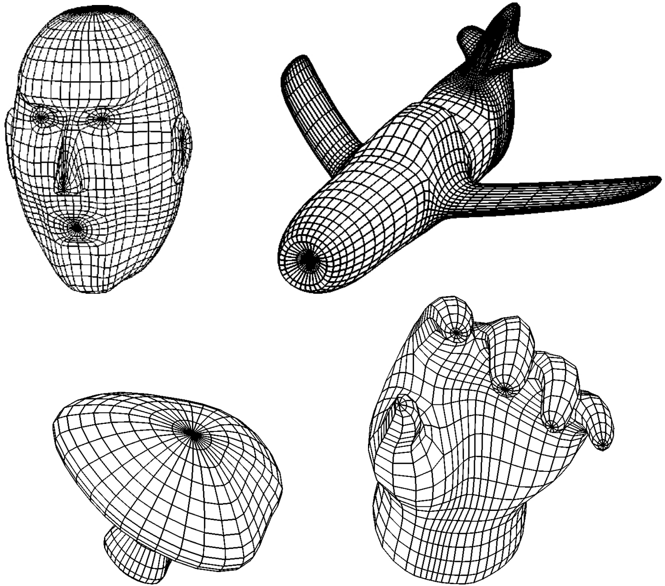
Polar is easily combined with Catmull-Clark



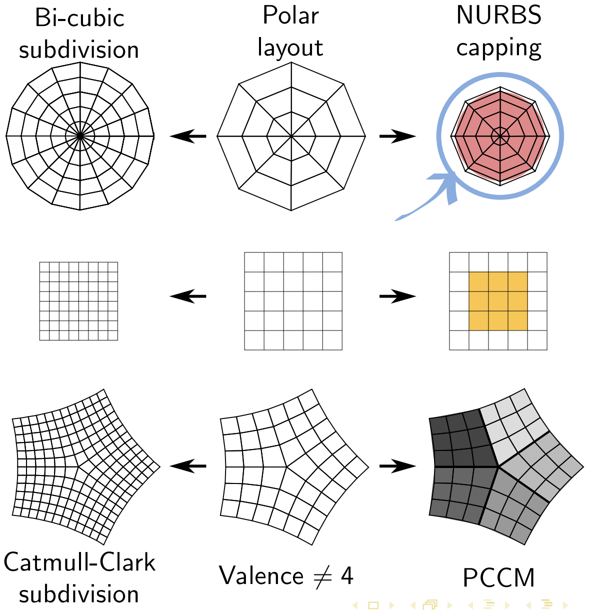
$\Rightarrow$ C^1 with bounded curvature at the polar limit point.

- Verified using standard analysis tools from subdivision theory.

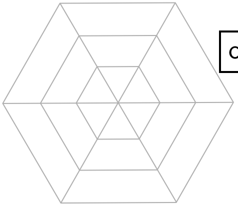
Results



NURBS Constructions

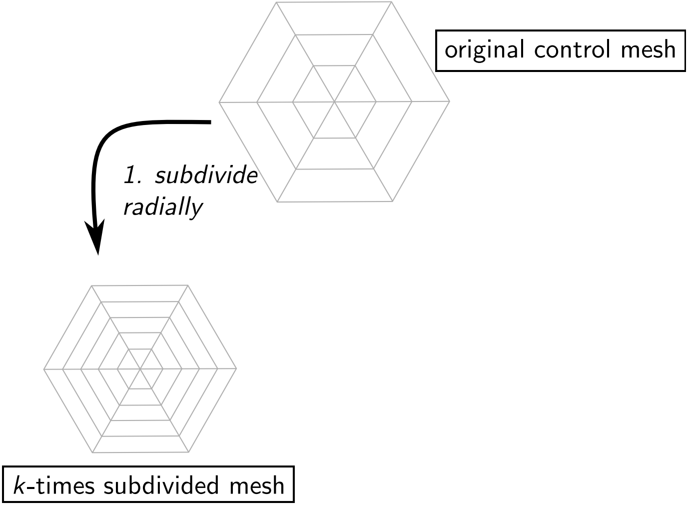


Polar structures can be C^1 capped by a single NURBS patch



original control mesh

Polar structures can be C^1 capped by a single NURBS patch



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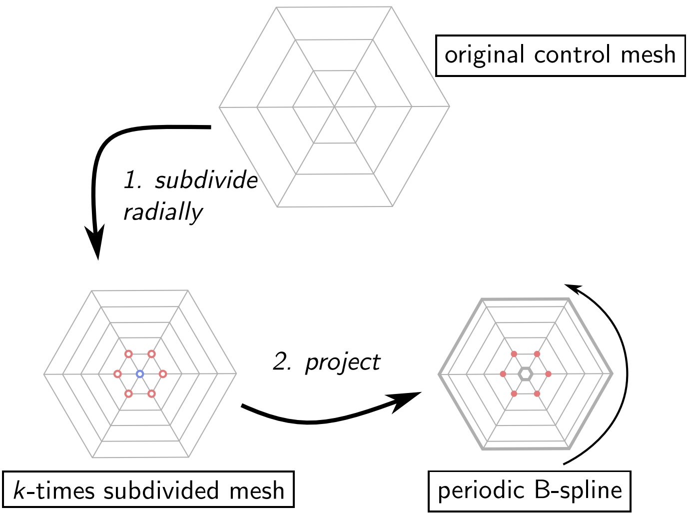
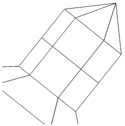


Capping Polar with a single NURBS patch



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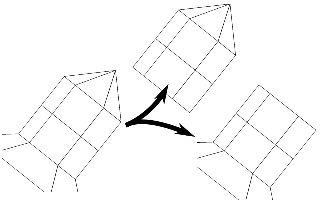

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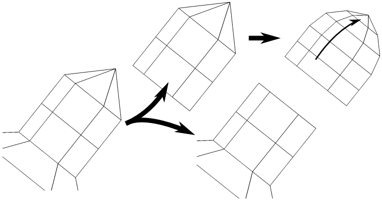

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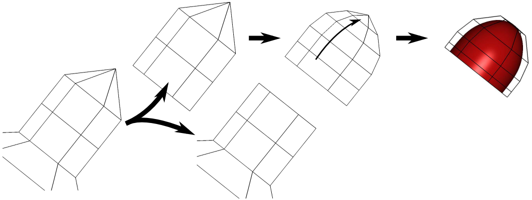

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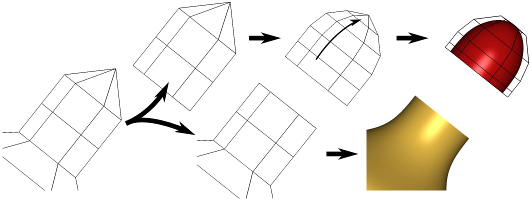

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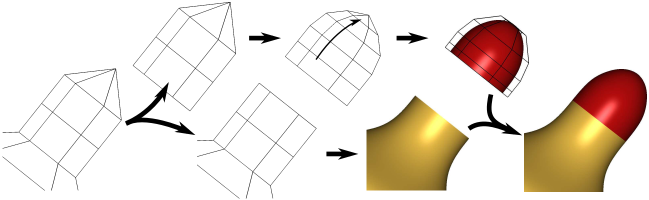

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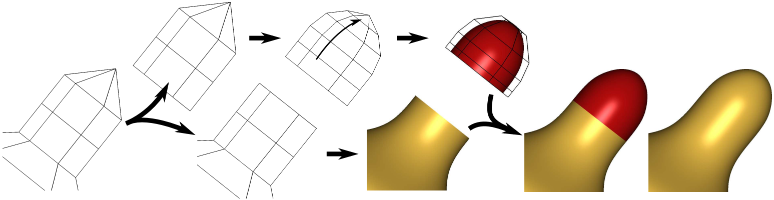

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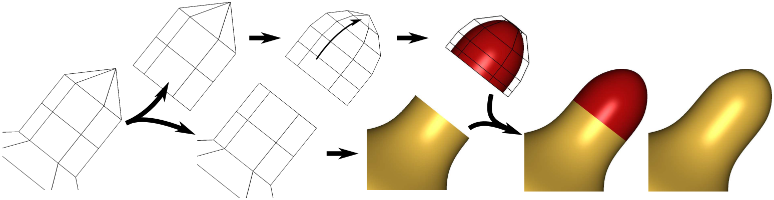
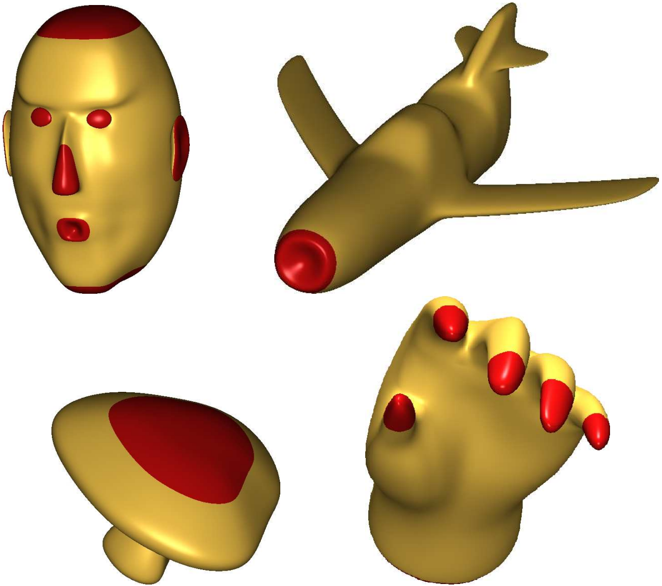


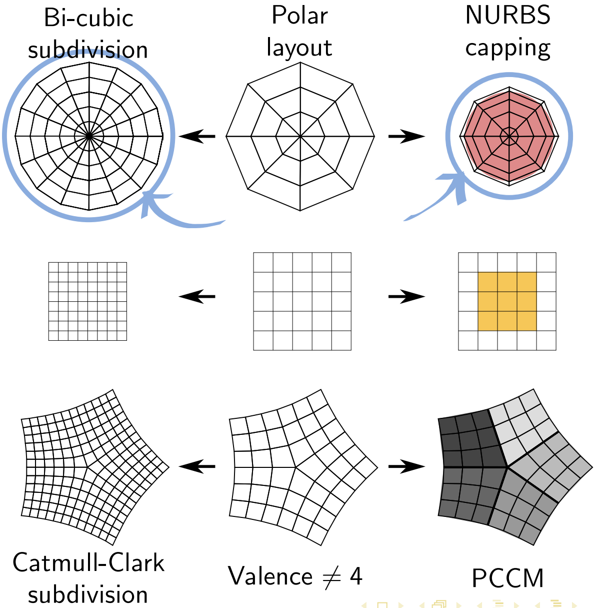
$\Rightarrow$ C^1 with bounded curvature at the polar limit point.

- Singular parametrizations typically tricky
- Our B-spline patch = limit surface of a particular subdivision scheme
- Analyze using subdivision machinery!

Results

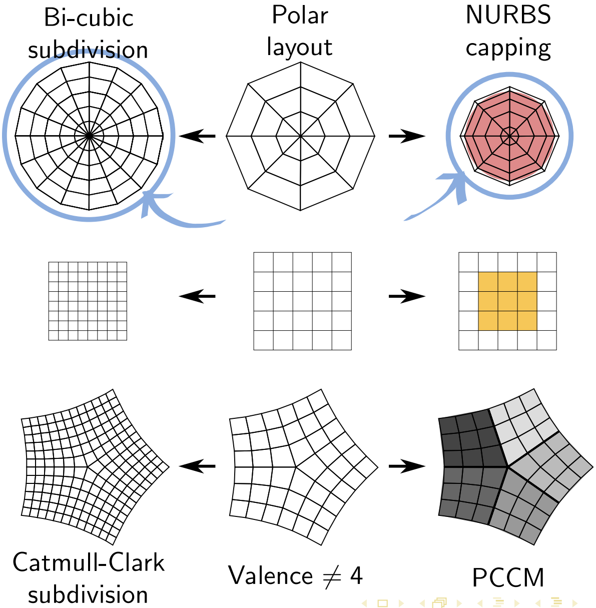


Conclusion



Bi-cubic subdivision
Polar layout
NURBS capping
Catmull-Clark subdivision
Valence ≠ 4
PCCM

Questions?



Bi-cubic subdivision
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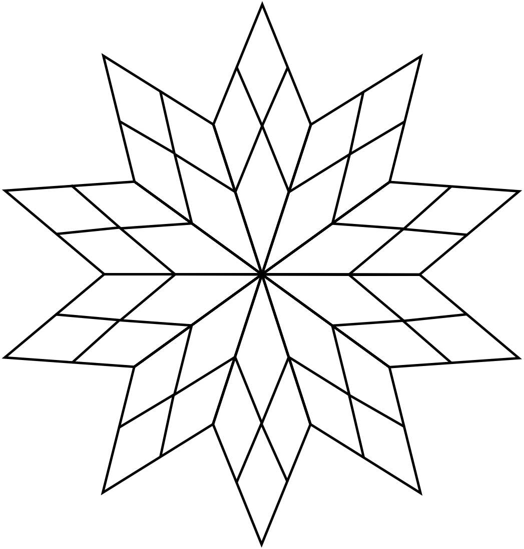
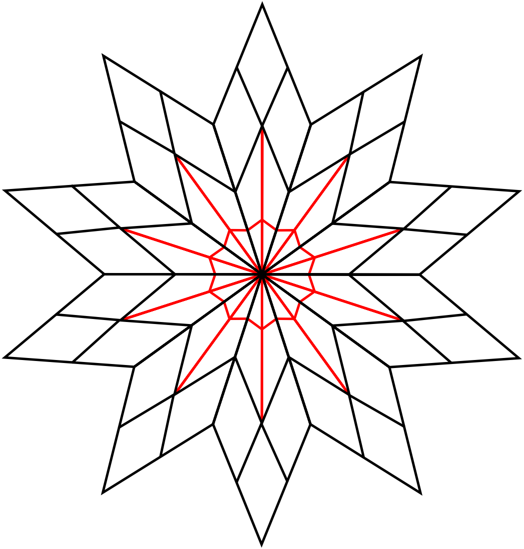
Backup Slides

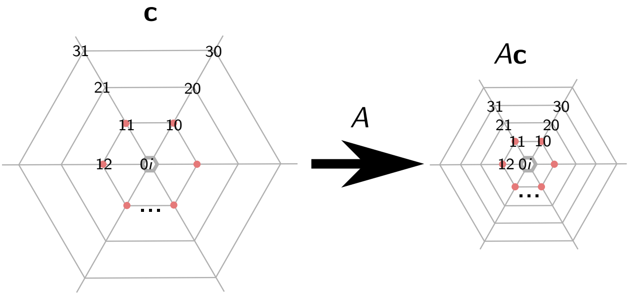
High-valent Catmull-Clark layout → polar layout



High-valent Catmull-Clark layout → polar layout



Analysis



$$A = \begin{bmatrix} A_0 & A_1 & \dots & A_{n-1} \\ A_{n-1} & A_0 & \dots & A_{n-2} \\ \vdots & & \ddots & \vdots \\ A_1 & \dots & A_{n-1} & A_0 \end{bmatrix}$$

$$A_0 := \begin{bmatrix} 1/n & 0 & 0 & 0 \\ 1/n & \Gamma_0 & 0 & 0 \\ 0 & 3/4 & 1/4 & 0 \\ 0 & 3/16 & 11/16 & 1/8 \end{bmatrix}, \quad A_i := \begin{bmatrix} 1/n & 0 & 0 & 0 \\ 1/n & \Gamma_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

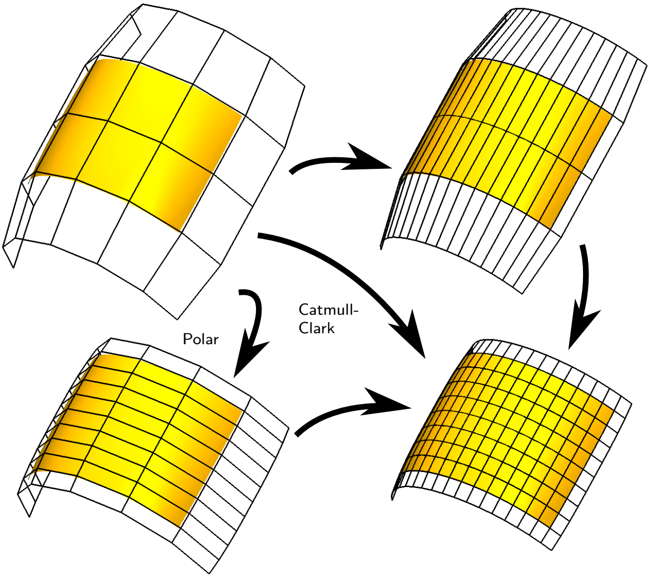
Analysis

Spectral analysis of A (circulant) gives desired properties:

- Eigenvalues: $1, \overbrace{1/2, 1/2}^{\substack{1^{st} \,\&\, n-1^{st} \\ \text{Fourier blocks}}}, \overbrace{1/4, \ldots, 1/4}^{n}.$
- Characteristic map is regular ($\Rightarrow C^1$).
- Geometric multiplicities = algebraic multiplicities for A.

$\Rightarrow$ Bounded curvature.

Tensor product B-spline refinement



Separating Catmull-Clark and polar extraordinary limit points